EXERCISE – II HINTS & SOLUTIONS

Sol.1 A,C

$y = x^2 + 6x + 10$ $\quad$ $y = ax^2 + bx + 7/2$

$$\frac{dy}{dx} = 2x + 6\Big|_{(-2,\,2)} = 2 \;;\; \frac{dy}{dx} = 2ax + b\Big|_{(1,\,2)}$$

$y - 2 = -2(x + 2)$ $\quad$ $\dfrac{dy}{dx} = 2a + b$

$y - 2 = -2x + 4$ $\quad$ $2a + b = -2$ ....(1)

$y + 2x = 6$ $\quad$ (1, 2) will lie on the curve

$2 = a + b + 7/2$

$a + b = -\dfrac{3}{2}$ ....(2)

from (1) & (2)

$a = 1,\ b = -5/2$

Sol.2 A,B

$$f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 7x - 4$$

$f'(x) = x^2 - 5x + 7\,|_P$ $\quad$ $P(x_1, y_1)$

$f'(x) = x_1^2 - 5x_1 + 7$

cuts equal intercepts that means slope = 1

$x_1^2 - 5x_1 + 7 = 1$

$x_1^2 - 5x_1 + 6 = 0$

$x_1 = 2\ ,\ x_1 = 3$

$y_1 = 8/3\ ,$ $\quad$ $y_1 = 7/2$

Point (2, 8/3) $\quad$ (3, 7/2)

Sol.3 A,D

$2y = x^2$ $\quad$ $P(x_1, y_1)$

$$\frac{dy}{dx} = \frac{2x}{2} = x\Big|_p = x_1$$

Equation of normal

$$y - y_1 = -\frac{1}{x_1}(x - x_1)$$

If will pass through (0, 3)

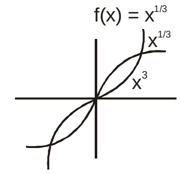
$$3 - y_1 = -\frac{1}{x_1}(0 - x_1)$$

$3 - y_1 = 1$ $\quad$ $x_1^2 = 4$

$y_1 = 2$ $\quad$ $x_1 = 2, -2$

point (2, 2), (–2, 2)

Sol.4 A,B,D



Tangent is y-axis at (0, 0)

$\Rightarrow\ x = 0$

Normal $y = 0$

Exactly three points

Sol.5 A,B

$y = \cos(x + y)$

$y' = -\sin(x + y)$ $\quad$ $(1 + y')$

$$y' = -\frac{\sin(x+y)}{1+\sin(x+y)} = -\frac{1}{2}$$

$\sin(x + y) = 1$

$\cos(x + y) = 0$

$y_1 = \cos(x_1 + y_1) = 0$

$\sin(x_1 + y_1) = 1 \Rightarrow \sin x_1 = 1$

$$x_1 = \frac{\pi}{2}, -\frac{3\pi}{2}$$

Point $\left(\dfrac{\pi}{2}, 0\right)$ and $\left(-\dfrac{3\pi}{2}, 0\right)$

This two points satisfies

Sol.6 A,C

$x = a(\cos\theta + \theta\sin\theta)$ $\quad$ $y = a(\sin\theta - \theta\cos\theta)$

$$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$$

$$\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$$

$$\frac{dy}{dx} = \frac{a\,\theta\sin\theta}{a\,\theta\cos\theta} = \tan\theta$$

Normal

$$y - a(\sin\theta - \theta\cos\theta) = -\frac{1}{\tan\theta}$$

$(x - a(\cos\theta + \theta\sin\theta))$ .....(1)

$y - a(\sin\theta - \theta\cos\theta) = -\cot\theta\,(x - a(\cos\theta + \theta\sin\theta))$

$$= \tan\left(\frac{\pi}{2} + \theta\right)(x - a(\cos\theta + \theta\sin\theta))$$

make are angle of $\left(\dfrac{\pi}{2} + \theta\right)$

Distance from origin of normal (1)

$$d = \left|\frac{-a(\sin\theta - \theta\cos\theta) - a(\cos\theta + \theta\sin\theta)\dfrac{\cos\theta}{\sin\theta}}{\sqrt{1 + 1/\tan^2\theta}}\right|$$

= a which is constant

Sol.7 A,B,C

$x = t^2 + 3t - 8$ $\quad$ $y = 2t^2 - 2t - 5$

at point (2, 1)

$2 = t^2 + 3t - 8$ $\quad$ $-1 = 2t^2 - 2t - 5$

$t = -5, 2$ $\quad$ $2t^2 - 2t - 4 = 0$

$t = 2$ $\quad$ $t = 2 \quad t = -1$

$x_1 = 4 + 6 - 8$ $\quad$ $y_1 = 8 - 4 - 5$

$x_1 = 2$ $\quad$ $y_1 = -1$

$\frac{dy}{dx} = 2t + 3$ $\quad$ $\frac{dy}{dt} = 4t - 2$

$$\frac{dy}{dx} = \left.\frac{4t-2}{2t+3}\right|_{t=2} = \frac{6}{7}$$

$m = 6/7$

$$L_T = \left|\frac{y_1\sqrt{1+m^2}}{m}\right| = \left|\frac{-1\sqrt{1+\frac{36}{49}}}{6/7}\right| = \left|\frac{\sqrt{85}}{6}\right|$$

Slope of tangent = 6/7

$$L_{ST} = \left|\frac{y_1}{m}\right| = \frac{7}{6}$$

Sol.8 A,B

$by = -ax - c$ $\quad$ $xy = 2$

$y = -\frac{a}{b}x - c$ $\quad$ $y + xy' = 0$

$y' = -\frac{y}{x}$

Slope of normal $= \frac{x}{y}$

$\frac{x_1^2}{2} = -\frac{a}{b}$ $\quad$ $= \frac{x_1}{y_1}$

$= \frac{x_1^2}{2} > 0$

LHS always positive

so RHS should be positive so a, b should have opposita sign

Sol.9 A,C

$\frac{x^2}{a^2} + \frac{y^2}{4} = 1$; $\quad$ $y^3 = 16x$

$\frac{2x}{a^2} + \frac{2yy'}{4} = 0$ $\quad$ $3y^2y' = 16$

$y' = \left.\frac{-4x}{a^2y}\right|_p$ $\quad$ $y' = \left.\frac{16}{3y^2}\right|_p$

$m_1 = -\frac{4x_1}{a^2y_1}$ $\quad$ $m_2 = y' = \frac{16}{3y_1^2}$

$m_1 \times m_2 = -1$

$$\frac{-4x_1}{a^2y_1} \times \frac{16}{3y_1^2} = -1$$

$$\frac{4}{3a^2} = 1$$

$$a^2 = \frac{4}{3}$$

$$a = \pm\frac{2}{\sqrt{3}}$$

Sol.10 A,C

$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ $\quad$ $(n \in N)$

$x = a \quad 1 + \left(\frac{y}{b}\right)^n = 2$

$y^n = b^n$

If n is even $\quad$ $y = \pm b$

If n is odd $\quad$ $y = b$

$n \to$ even (a, b) & (a, –b)

$n \to$ odd (a, b)

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$$

$$n\left(\frac{x}{a}\right)^{n-1}\left(\frac{1}{a}\right) + n\left(\frac{y}{b}\right)^{n-1}\left(\frac{1}{b}\right)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^n}{a^n}\,\frac{x^{n-1}}{y^{n-1}}$$

$n \to$ even

$$(a, b) \Rightarrow \frac{dy}{dx} = -\left(\frac{b}{a}\right)^n\left(\frac{a}{b}\right)^{n-1} = -\frac{b}{a} = m_T$$

$M_N = \frac{a}{b}$

$y - b = \frac{a}{b}(x - a)$

$by - b^2 = ax - a^2$

$ax - by = a^2 - b^2$

$(a, -b)\ \dfrac{dy}{dx} = -\left(\dfrac{b}{a}\right)^n \left(-\dfrac{a}{b}\right)^{n-1} = \dfrac{b}{a} = m_T$

$M_N = -a/b$

$y + b = -\dfrac{a}{b}(x - a)$

$ax + by = a^2 - b^2$

Sol.11 A,D

$y = x^2 + ax + b$ (1, 0) satisfies

$y = x(c - x)$ $1 + a + b = 0$

$= cx - x^2$ $c - 1 = 0 \Rightarrow c = 1$

$m_1 = \dfrac{dy}{dx} = 2x + a|_{(1,0)} = 2 + a$

$m_2 = \dfrac{dy}{dx}\ c - 2x|_{(1,0)} = c - 2$

$2 + a = c - 2$

$c = 1 \quad a = -3$

$a + b = -1$

$b = -1 - a$

$b = -1 + 3 = 2$

$b + c = 3$

Sol.12 A,B

$x = 2 \ln \cot t + 1$ $y = \tan t + \cot t$

$\dfrac{dx}{dt} = \dfrac{2}{\cot t}(-\operatorname{cosec}^2 t)$ $\dfrac{dy}{dt} = \sec^2 t - \operatorname{cosec}^2 t$

at $t = \dfrac{\pi}{4}$

$\dfrac{dx}{dt} = -\dfrac{2}{\sin^2 t} \tan t$ $\dfrac{dy}{dt} = 2 - 2 = 0$

$= \dfrac{-2}{(1/2)} = -4$

$\dfrac{dy}{dx} = 0$ Tangent is parallel to x-axis

Normal is parallel to y-axis

Sol.13 B,C

$y = ke^{kx}$ at y-axis

$m = \dfrac{dy}{dx} = k^2 e^{kx}$ $x = 0, y = k$

$m = k^2$

$\tan\theta = \dfrac{1}{k^2}$

$\theta = \tan^{-1}\left(\dfrac{1}{k^2}\right) = \cot^{-1} k^2 = \sec^{-1}\left(\dfrac{1}{\sqrt{1+k^4}}\right)$

Sol.14 A,C,D

(A) $y^2 = 4ax$ $y = e^{-x/2a}$

$m_1 = \dfrac{4a}{2y} = \dfrac{2a}{y_1}$; $m_2 = -\dfrac{1}{2a} e^{-x/29}$

$= -\dfrac{1}{2a} y_1$

$m_1 \times m_2 = -1$

(B) $y^2 = 4ax$ $x^2 = 4ay$

$y' = \dfrac{4a}{2y}$ $2x = 4ay'$

$y' = \dfrac{2x}{4a}$

(C) $xy = a^2$ $x^2 - y^2 = b^2$

$y + xy' = 0$ $2x - 2yy' = 0$

$y' = -\dfrac{y}{x}$ $y' = \dfrac{x}{y}$

$m_1 \times m_2 = -1$

(D) $y = ax$ $x^2 + y^2 = c^2$

$y' = a$ $2x + 2yy' = 0$

$m_1 \times m_2 = -\dfrac{ax}{y} = -1$ $y' = -\dfrac{x}{y}$

Sol.15 A,C

$y = f(x)$

Let $f(x) = ax^2 + bx + c$

this parabola touches $y = x$ line at (1, 1)

that means slope at (1, 1) = 1

$f'(x) = 2ax + b$

$2a + b = 1$ ....(1)

(1, 1) will also satisfied the curve

$1 = a + b + c$ ....(2)

$f'(1) = 1$

$2f(0) = 1 - f'(0)$

$f(0) = c$

LHS = 2c

RHS = $1 - f'(0)$ from (1) & (2) $a = c$

$= 1 - b$

$= 2a \quad = 2c$